Foundation of Cryptography (0368-4162-01), Lecture 9
Secure Multiparty Computation

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Section 1

The Model

Multiparty Computation

- Multiparty Computation – computing a functionality f

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- Multiparty Computation – computing a functionality *f*
- Secure Multiparty Computation: compute *f* in a "secure manner"

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  Real Vs. Ideal model

The Model   Oblivious Transfer   Yao Garbled Circuit

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Real Model Execution

Let $\overline{A} = (A_1, A_2)$ be a pair of algorithms, and $x_1, x_2 \in \{0, 1\}^*$.
Define $\text{REAL}_{\overline{A}}(x_1, x_2)$ as the joint outputs of $(A_1(x_1), A_2(x_2))$

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- A *semi-honest* party follows the protocol, but might output additional information

Ideal Model Execution

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Let $\overline{B} = (B_1, B_2)$ be a pair of oracle-aided algorithms. An execution of $\overline{B}$ in the ideal model on inputs $x_1, x_2 \in \{0, 1\}^*$, denoted $\text{IDEAL}_{f, \overline{B}}(x_1, x_2)$, is the joint output of the parties in the end of the following experiment:

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1. The input of B_i is x_i ($i \in \{0, 1\}$)

2. Each party sends value y_i to the *trusted party* (possibly $\perp$)

3. Trusted party sends $f_i(y_0, y_1)$ to B_i (sends $\perp$, if $\perp \in \{y_0, y_1\}$)

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A protocol π securely computes f, if $\forall$ real model, admissible PPT $\overline{A} = (A_1, A_2)$, exists an ideal-model admissible pair PPT $\overline{B} = (B_1, B_2)$, s.t.

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where the enumeration is over all $x_1, x_2 \in \{0, 1\}^*$ with $|x_1| = |x_2|$.

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- Auxiliary inputs
- Security parameter
- We focus on semi-honest adversaries

Section 2

Oblivious Transfer

Oblivious Transfer

A protocol that securely realizing the functionality
OT: $(\{0,1\}^* \times \{0,1\}^*) \times \{0,1\} \mapsto \{0,1\}^* \times \bot$, where $f_1(\cdot) = \bot$
and $f_2((\sigma_0, \sigma_1), i) = \sigma_i$ and .

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- "Complete" for multiparty computation
- We show how to construct for bit inputs

Oblivious Transfer from Trapdoor Permutations

Let (G, f, Inv) be a TDP and let b be an hardcore predicate for f.

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Protocol 2 ((S, R))

Common input: 1^n, S**'s input:** $\sigma_0, \sigma_1 \in \{0, 1\}$, R**'s input:** $i \in \{0, 1\}$

1. S chooses $(e, d) \leftarrow G(1^n)$, and sends e to R

2. R chooses $x_0, x_1 \leftarrow \{0, 1\}^n$, sets $y_i = f_e(x_i)$ and $y_{1-i} = x_{1-i}$, and sends y_0, y_1 to S

3. S sets $c_j = b(\mathsf{Inv}_d(y_j)) \oplus \sigma_j$, for $j \in \{0, 1\}$, and sends (c_0, c_1) to R

4. R outputs $c_i \oplus b(x_i)$.

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Claim 3

Protocol 2 securely realizes OT (in the semi-honest model).

Proving Claim 3

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- Secrecy: We need to prove that $\forall$ real model, semi-honest, admissible PPT $\overline{A} = (A_1, A_2)$, exists an ideal-model, admissible pair PPT $\overline{B} = (B_1, B_2)$ s.t.

$$\{\mathsf{REAL}_{\overline{A}}(1^n, (\sigma_0, \sigma_1), i\} \approx_c \{\mathsf{IDEAL}_{\mathsf{OT}, \overline{B}}(1^n, (\sigma_0, \sigma_1), i\}, \quad (1)$$

where the enumeration is over $n \in \mathbb{N}$ and $\sigma_0, \sigma_1, i \in \{0, 1\}$

R's privacy

For $\overline{A} = (S', R)$, where S' is a semi-honest implementation of S, let $\overline{B} = (S'_{\mathcal{I}}, R_{\mathcal{I}})$ be an ideal-model protocol, where $R_{\mathcal{I}}$ acts honestly, and

Algorithm 4 ($S'_{\mathcal{I}}$)

input: $1^n, \sigma_0, \sigma_1$

1. Send (σ_0, σ_1) to the trusted party
2. Emulate $S'(1^n, \sigma_0, \sigma_1)$, acting as $R(1^n, 0)$
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Algorithm 6 ($R'_{\mathcal{I}}$)

input: $1^n, i \in \{0, 1\}$

1. Send i to the trusted party, and let σ be its answer.
2. Emulate $R'(1^n, i)$, acting as $S(1^n, \sigma_0, \sigma_1)$, where $\sigma_i = \sigma$, and $\sigma_{1-i} = 0$
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Section 3

Yao Garbled Circuit

Before we start

- Fix a (multiple message) semantically-secure private-key encryption scheme (G', E, D) with

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  2. $D_d(E_{d'}(m)) = \perp$, for any $d \neq d'$ and $m \in \{0, 1\}^*$.

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- Boolean circuits: gates, wires, inputs, outputs, values, computation

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- For any $w \in \mathcal{W}$, associate two random 'keys" $k_0^w, k_w^1 \in \{0,1\}^n$.
- For $g \in G$ with input wires i, j and output wire h, let $T(g)$ be the following table

input wire i	input wire j	output wire h	hidden output wire
k_i^0	k_j^0	$k_h^{g(0,0)}$	$E_{k_i^0}(E_{k_j^0}(k_h^{g(0,0)}))$
k_i^0	k_j^1	$k_h^{g(1,0)}$	$E_{k_i^0}(E_{k_j^1}(k_h^{g(0,1)}))$
k_i^1	k_j^0	$k_h^{g(0,1)}$	$E_{k_i^1}(E_{k_j^0}(k_h^{g(1,0)}))$
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Figure: Table for gate g, with input wires i and j, and output wire h.

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- Given $\widetilde{T} = \{(g, \widetilde{T}(g))\}_{g \in G}$, $\{k_i^{C(x)_w}\}_{w \in \mathcal{I}}$, for some $x \in \{0,1\}^\ell$, and $\{(w, k_w)\}_{w \in \mathcal{O}}$, we can efficiently compute $C(x)$

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- Given $\widetilde{T} = \{(g, \widetilde{T}(g))\}_{g \in G}$, $\{k_i^{C(x)_w}\}_{w \in \mathcal{I}}$, for some $x \in \{0,1\}^{\ell}$, and $\{(w, k_w)\}_{w \in \mathcal{O}}$, we can efficiently compute $C(x)$
- No other information about x leaks!
- Can we use garbled circuit for secure computation?

The protocol

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Protocol 8 ((A, B))

Common input: 1^n. A/B**'s input:** $x_1/x_2 \in \{0,1\}^{\ell}$

1. A prepares random $\{k_w = (k_w^0, k_w^1)\}_{w \in \mathcal{W}}$ and $\widetilde{T}$, and sends $\widetilde{T}$, $\{(w, k_w^{C(x_1, \cdot)_w})\}_{w \in \mathcal{I}_1}$ and $\{(w, k_w)\}_{w \in \mathcal{O}_2}$ to B.

2. $\forall w \in \mathcal{I}_2$, A and B interact in $(S(k_w), R(C(\cdot, x_2)_w))(1^n)$.

3. B computes the (garbled) circuit, and sends $\{(w, k_w^{C_{x_1, x_2}[w]})\}_{w \in \mathcal{O}_2}$ to A.

4. The parties compute $f(x_1, x_2)_1$ and $f(x_1, x_2)_2$ respectively.

Claim 9

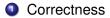Protocol 8 securely computes f (in the semi-honest model)

Claim 9

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Proof:

1 Correctness

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1. Correctness
2. B's privacy

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   Immediately follows from the security of the OT
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Proof:

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   Immediately follows from the security of the OT

3. A's privacy
   The simulator for B puts random values in $\widetilde{T}$,
   $\{(w, k_w^{C(x_1, \cdot)_w})\}_{w \in \mathcal{I}_1}$ and $\{(w, k_w^{C(\cdot, x_2)_w})\}_{w \in \mathcal{I}_1}$, and sets
   $\{(w, k_w)\}_{w \in \mathcal{O}_2}$ according to $f_2(x_1, x_2)$.

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Course Summary

See diagram

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- and....